LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION - **PHYSICS**

SECOND SEMESTER – **APRIL 2013**

PH 2812 - MATHEMATICAL PHYSICS

Date : 02/05/2013 Dept. No. Max. : 100 Marks
Time : 9:00 - 12:00

PART A

Answer **ALL** questions 10 x 2 = 20

1) Check whether f(z) = x-iy is analytic or not by Cauchy-Riemann equations.
2) Find the Laurent series of $(\sin z)/z^5$ about z= 0, upto five terms
3) Define the Dirac delta function. What is its Laplace transform?
4) Evaluate the Fourier cosine transform of the first derivative of f (x).
5) What are the possible two initial conditions in the vibration of a rectangular membrane? Explain the symbols used.
6) Solve $\frac{\partial^2 u(x,y)}{\partial x^2} - u\,(x,y) = 0$.
7) State the orthonormality property of the Legendre polynomials.
8) Evaluate the 2nd order Hermite polynomial by Rodrigue’s formula.
9) List the four properties for a set of elements to form a group.
10) Distinguish Abelian group from cyclic group.

PART B

Answer any **FOUR** questions 4 x 7.5 = 30

11) Show that the function v (x,y) = -sin x sinh y is harmonic. Construct the corresponding analytic function f(z) = u (x, y) + i v (x, y).
12) Solve the initial value problem $\frac{d^2y}{dt^2} + 25y = 10\cos 5t$, y (0) = 2, $\frac{dy(0)}{dt} = 0$ by the Laplace transforms .
13) Using the method of separation of variables, solve the partial differential equation $\partial u/\partial x + \partial u/\partial y = (x+y)\,u$.
14) Assuming the recurrence formulae (i) $d/dx\,[x^n J_n(x)] = x^n J_{n-1}(x)$ and (ii) $d/dx\,[x^{-n} J_n(x)] = -x^{-n} J_{n+1}(x)$, establish the $J_{3/2}(x) = (1/x)\,J_{1/2}(x) - J_{-1/2}(x)$, where $J_n(x)$ is the Bessel function of the first kind.
15) Work out the multiplication table of the symmetry group of the proper covering operations of an equilateral triangle. Write down all the subgroups and classes .

PART C

Answer any **FOUR** questions 4 x 12.5 = 50

16) Using the contour integration, evaluate the following real integral. $\int_0^{2\pi} \frac{1+2\cos\theta}{5+4\cos\theta}\, d\theta$.

17) (a) Find the Fourier transform of (i) $f(x) = \exp(-x^2)$. (b) Represent $f(t) = \sin 2t$, $2\pi < t < 4\pi$ and $f(t) = 0$ otherwise, in terms of unit step function and find its Laplace transform.

18) Solve the one dimensional heat equation $\partial u/\partial t = c^2\, \partial^2 u/\partial x^2$ where $u(x,t)$ is the temperature in a body of homogeneous material with the boundary conditions $u(0,t) = 0$ and $u(L,t) = 0$ for all t and the initial condition(initial temperature) as $u(x,0) = f(x)$. Solve it by the method of separation of variables and use of Fourier series.

19) Solve the Hermite differential equation $d^2y/dx^2 - 2x\, dy/dx + 2ny = 0$ by the power series method.

20) (a) Obtain the transformation matrices of the symmetry elements (i) for the axis of symmetry and (ii) for the improper axis of symmetry .

(b)Enumerate and explain the symmetry elements of H_2O and NH_3 molecules.

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